Leadership in Public Projects: Vagueness or Clarity

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Abstract

Even if it is costless to inform everybody about the quality of a project, it may improve cooperation to inform only one player and appoint him to be the leader. One key assumption of the model is that the leader can **only partially** transmit his information to the others via a costly commitment signal. A credible leader can improve cooperation by sending a vague (rather than a precise) signal to his followers. Partial revelation of information then supports cooperation more effectively on average than does full revelation or complete information. A credible leader acts as a representative player on behalf of the group and improves the welfare **ex-post** if his commitment cost is at least as large as the contribution cost of the others. He improves the welfare on average **(ex-ante)** even if he is not a representative player. The last two results are shown for linear utility functions.

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1 Introduction

The private provision of public goods is an area of much study in public economics. Several researchers have examined the private provision of public goods in simultaneous-move Nash games in which all agents choose their contribution levels without knowledge of others' contribution decisions. A standard result of this theoretical research is that pure public goods are under-provided by voluntary contributions of private individuals. For details see Bergstrom, Blume and Varian (1985), and Corns and Sandler (1984,85).

Varian (1992) addresses the role of leadership in public projects by considering a Stackelberg contribution game in which players sequentially contribute to a public project with an additive nature. In his model, the leader's move is observable to the followers and therefore the leader can credibly commit to his contribution in a way that is not possible in a simultaneous move game. The main finding of Varian is that the total contribution in sequential-move games does not exceed that in a simultaneous-move game. The reason is that the first mover can free ride on his follower by committing to a low initial donation.

Leaders, however, increase the overall contribution in some public projects. Sung (1998) shows that Varian's result is reversed for weakest link public goods[1] and does not necessarily hold for best shot public projects[2]. Romano and Yildirim (1998) show that sequential-move contributions lead to larger donations if utility functions are increasing in donations and the followers' best response function is increasing in the contribution of the leader.

The closest antecedents to my work are Hermalin (1998), Vesterlund (2003) and Andreoni (2004). Hermalin investigates a team production problem in which only

[1] The weakest link public good has its aggregate supply level defined as the minimum of all individual contribution levels.

[2] The best shot public good, has a social composition level being equal to the maximum of all individual contributions. In other words, only the best counts.

one player (the leader) is exogenously informed about the marginal return to effort. Hermalin's leader puts in long hours on the organizational activities to convince his followers to put in more effort. The leader's action fully reveals his information about the rate of return to effort and increases the overall contribution above the contribution level of complete information scenario. Vesterlund (2003) applies Hermalin's insight to a model of charitable contributions, but in her model the leader endogenously decides whether to acquire information before deciding whether to contribute. Vesterlund focuses on whether a third player, a fund-raiser who moves first, chooses to announce in equilibrium that the leader's contribution will be public. Andreoni (2004) builds on Vesterlund's (2003) model by endogenizing the selection of the leader. In Andreoni's model, agents with low costs of contribution (e.g., the rich) will step forward to accept this costly role. Vesterlund's and Andreoni's leaders, like Hermalin's, fully reveal their information.

The above models by Vesterlund, Hermalin and Andreoni are interesting and produce some counter-intuitive conclusions. The main theme of these papers is that the leader is able to induce higher participation by fully transmitting his information to the other players via a welfare increasing signal.

This paper changes the above theme in the following way. First, in my model the leader's signal is not productive by itself. Second, the leader is **unable to credibly reveal all of his information** to the others through his signal. A leader may be unable to reveal his information fully and credibly for different reasons. One likely reason is the complexity of the information. An expert leader who has access to complex information may be practically unable to transfer all of his information to others for the information is too complex to be fully understood. Such a situation can also occur when the information is not complicated but difficult to verify. For example, much information is revealed by a presidential candidate which is not fully verifiable by a potential voter. A presidential candidate therefore is unable to credibly transmit all of his information to the voters. The same situation occurs when a person endorses a public project (a person's endorsement is a positive signal of the quality of a public project but does not reveal the project's exact rate of return). I argue that on average ill-informed followers tend to be more cooperative when the leader is unable to credibly reveal all of his information because they do not know when their cooperative actions actually produce high personal payoffs.

I consider m+1 identical players who decide how to allocate their endowment between a public project and their private consumption. Their utility function is quasi-linear: linear in the aggregate contribution to the public project and strictly concave in private consumption[3].

Two different scenarios are considered. In the first scenario all players are informed about the quality of the public project and simultaneously decide how much to contribute to it. In the second scenario only one player (the leader) is informed about the quality of the project and the others are uninformed followers. I consider a two stage sequential move game. The first mover (the leader) has two strategies. First, he decides whether to make a costly commitment to the public project and then he decides how much to contribute. Followers decide only how much to contribute to the public project. A key assumption is that the second movers (the followers) observe only the leader's commitment signal but are unable to verify his exact amount of contribution. This prevents full revelation and allows the leader to persuade his followers to cooperate in cases where they would refuse to cooperate if they were fully informed.

I show that for lower value projects partial revelation of information induces more contribution **ex-post** than full revelation or complete information if the leader is credible[4]. For higher value projects, however, partial revelation may (or may not) create higher contribution **ex-post** than full revelation or complete information.

The ex-ante result favors the partial revelation scenario more strongly. I show that partial revelation of information creates more expected contribution **ex-ante** than full revelation or complete information if the leader is credible. In other words,

[3] It is more general to consider a concave utility function. Considering a concave utility function, however, introduces a crowding out effect, in the sense that the contribution of one player crowds out the contribution of the rest. This introduces more complexity but no new phenomenon of interest.

[4] A credible leader is a leader who can convince his followers that the public project produces high personal payoffs. The leader's credibility will be explained in further detail as the analysis continues.

a credible leader increases contributions on average by sending a vague rather than a precise signal.

The leader's signal has two separate effects on the total surplus obtained by the group. First, it reduces the total surplus for it is costly but unproductive by itself. Its secondary effect, however, is to increase the total surplus by pursuing cooperation.

I derive efficiency results for linear utility functions and show that a credible leader improves efficiency on average by partially revealing his information to the others.

This paper is organized as follows. Section 2 introduces a public good game under complete information which exhibits the standard free-riding problem. Section 3 introduces the second scenario where an informed leader leads a group of uninformed followers by partial revelation of his information. Section 4 shows how the leader affects cooperation and compares it with the case where the information is complete or fully revealed. Section 5 derives the efficiency results for linear utility functions. In section 6, I conclude the paper and discuss further developments.

2 Model 1: Provision of Public Goods under Complete Information

In this section, I develop a benchmark model of public goods provision under complete information. Consider $m+1$ identical players $i \in I = \{0, 1, 2, ..., m\}$. Each player divides his endowment w between consumption of a private good, $y_i \geq 0$, and contribution to a public project, $x_i \geq 0$. Therefore, $x_i = w - y_i$.

The utility function of each player has the following form:

$$V(x_0, x_1, ..., x_m) = \alpha \sum_{j=0}^{m} x_j + U(w - x_i)$$

where α is the marginal return to the aggregate contribution to the public project. I assume that α is distributed on the interval $[0, \overline{\alpha}]$ with the strictly positive density function $f(\alpha)$. I also assume that $U : \Re_+ \longrightarrow \Re$ is a C^3 function, which is strictly concave ($U'' < 0$) and strictly increasing ($U' > 0$) over the interval $[0, w]$. Further-more, I assume that $\underset{y\rightarrow 0}{Lim} U'(y) = +\infty$, $\underset{y\rightarrow \infty}{Lim}\ U'(y) = 0$[5] and players' absolute risk aversion is non increasing, implying $U''' > 0$[6]. Finally, I assume that $U'(w) < \overline{\alpha}$. This assumption implies that players are willing to contribute to the project for some values of $\alpha < \overline{\alpha}$.

I consider a simultaneous move game: α is determined by the nature, all players observe α and then simultaneously decide how much to contribute to the public project.

Consider the maximization problem of a representative player. Define $\widetilde{\alpha} = U'(w)$, and Let $\widetilde{X}(\alpha)$ be the optimal contribution of a representative player. Clearly, for $\alpha > \widetilde{\alpha}$ each player's optimal contribution level is strictly positive ($\widetilde{X}(\alpha) > 0$) and (given the assumptions) is the unique solution to the following first order condition:

$$\alpha = \frac{\partial U(w - x_i)}{\partial x_i}$$

For $\alpha \leq \widetilde{\alpha}$, players optimal contribution is zero ($\widetilde{X}(\alpha) = 0$).

The unique Nash equilibrium of this game is the symmetric strategy profile ($\widetilde{X}(\alpha)$). As previously stated contributions are zero at the equilibrium if $\alpha \leq \widetilde{\alpha}$ and strictly positive if $\alpha > \widetilde{\alpha}$.

One motivation for my work is the observation that players may refuse to make a positive contribution even when it is efficient to do so. To see this define

$$\Delta W(\alpha) = (m+1)\left[\alpha(m+1)X(\alpha) + U(w - X(\alpha)) - U(w)\right]$$

to be the welfare gain obtained by the group if all players contribute $X(\alpha) > 0$. The free riding problem occurs when $\Delta W(\alpha) > 0$ for some $\alpha \leq \widetilde{\alpha}$, because players refuse to contribute to the project at the equilibrium while efficiency requires positive contributions.

In the next section, I show that a leader who is given exclusive information about α can increase contributions via an initial endorsement which partially reveals his

[5] The limit assumptions are only technical assumptions to ensure the existence of an interior solution.

[6] Player i's absolute risk aversion is non increasing if $\frac{-U'''U' + U''^2}{U'^2} \leq 0$. Since $U' > 0$ by assumption, U''' should be positive for the inequality to hold.

information to the others.

3 Model 2: Leadership and the Provision of Public Goods

This section pursues the idea that a credible leader who has exclusive information about α can increase contributions via partial (rather than full) revelation of his information. That is, a leader can increase contributions by sending a vague rather than a precise persuasive signal.

I consider the model from section 2 but revise the timing and the information structure of the game. In the new scenario, α is observed only by one player (the leader). The distribution of α is assumed to be common knowledge.

In the first stage of the game, the leader makes two decisions: whether to commit and how much to contribute to the project. The leader's commitment strategy is $C : [0, \overline{\alpha}] \longrightarrow \{0, 1\}$. The value of $C(\alpha)$ is equal to 1 if the leader makes a commitment to the public project and 0 if he does not. The leader's contribution strategy is $X_0 : [0, \overline{\alpha}] \longrightarrow \Re_+$. The value of $X_0(\alpha)$ is 0 if the leader does not contribute to the project and is a positive number if he does. The leader's commitment is assumed to be costly[7]. The commitment cost $R : [0, \overline{\alpha}] \longrightarrow \Re_+$ is specified as:

$$R(\alpha) = m\theta C(\alpha) r(\alpha),$$

where $r : [0, \overline{\alpha}] \longrightarrow \Re_+$ is a reputation loss function, and $\theta > 0$ is an exogenous scaling factor.

I assume that the reputation loss function is decreasing in α ($r' < 0$), meaning that a leader who commits to a higher quality project is less likely to lose his reputation. I also assume that $r(\alpha)$ is a convex function of α if $\alpha \leq \widetilde{\alpha}$ and is equal to zero if $\alpha > \widetilde{\alpha}$. That is, the leader does not lose his reputation for committing to high value projects to which fully informed individuals are willing to contribute.

[7] One might think of it as the reputation that the leader loses if he makes a commitment to a low return project.

At the end of the first stage followers observe the leader's commitment but are unable to observe his amount of contribution. This is a key assumption which prevents full revelation of the information.

In the second stage of the game, having observed the leader's action, followers update their beliefs about α and simultaneously decide how much to contribute to the public project. A follower's strategy is $X_f : \{0,1\} \longrightarrow \Re_+$.

The leader's and followers' utility functions are

$$V_0(x_0, x_1, ..., x_m) = \alpha \sum_{j=0}^{m} x_j + U(w - x_0) - R(\alpha)$$

and

$$V_f(x_0, x_1, ..., x_m) = \alpha \sum_{j=0}^{m} x_j + U(w - x_f)$$

respectively.

Let A be a measurable subset of $[0, \overline{\alpha}]$ and $\mu(A) = prob(\alpha \in A)$. Given $C(\alpha)$, define $A_0 = \{\alpha \in [0, \overline{\alpha}] : C(\alpha) = 0\}$ and $A_1 = \{\alpha \in [0, \overline{\alpha}] : C(\alpha) = 1\}$.

A pure strategy perfect Bayesian equilibrium of the game is a strategy profile $\left(X_0^*, X_f^*, C^*\right)$ and the posterior beliefs $\mu(A \mid A_0)$ and $\mu(A \mid A_1)$ such that:

$$X_0^*(\alpha) = Arg \max_{x_0} \ \alpha \left[x_0 + mX_f^*(C^*(\alpha))\right] + U(w - x_0), \forall \alpha \tag{1a}$$

$$\mathrm{X}_f^* \left(C^*(\alpha)\right) = Arg \max_{x_f} \left\{ \underset{\alpha'}{E} \left[\alpha' \left(X_0^*(\alpha') + x_f + \sum_{j \neq f} X_j(C^*(\alpha)) \right) \mid C^*(\alpha) \right] + \mathrm{U}\left(\mathrm{w}\text{-}x_f\right) \right\}, \forall \alpha \tag{1b}$$

$$\begin{aligned} C^*(\alpha) &= 1 \quad if \quad \alpha m \left[X_f^*(1) - X_f^*(0)\right] - m\theta r(\alpha) > 0 \\ C^*(\alpha) &= 0 \quad if \quad \alpha m \left[X_f^*(1) - X_f^*(0)\right] - m\theta r(\alpha) < 0 \end{aligned} \tag{1c}$$

$$\mu(A \mid A_0) = \frac{\mu(A)\mu(A_0 \mid A)}{\mu(A_0)} \qquad for\ all\ measurable\ A \in [0, \overline{\alpha}] \tag{1d}$$

$$\mu(A \mid A_1) = \frac{\mu(A)\mu(A_1 \mid A)}{\mu(A_1)} \qquad for\ all\ measurable\ A \in [0, \overline{\alpha}] \tag{1e}$$

Note: I restrict my attention to perfect Bayesian equilibria such that the events A_0 and A_1 happen with positive probability.

Equation $1a$ says that the leader's optimal contribution level $X_0^*(\alpha)$ maximizes his utility for all possible values of α. Equations $1b$ and $1c$ are the perfection conditions. Equation $1b$ determines the follower's optimal contribution level $X_f^*(C^*(\alpha))$. It states that followers react optimally to the leader's action given their posterior beliefs about α. The leader's optimal commitment strategy $C^*(\alpha)$ is determined by $1c$, which states that the leader takes into account the effect of his commitment on his followers' amount of contribution. Finally equations $1d$ and $1e$ correspond to the application of Bayes' rule.

To help build intuition about the equilibrium conditions I analyze the game in some detail. Recall that in the first stage of the game, only the leader observes the exact value of α. After observing α the leader decides whether to commit and how much to contribute to the public project. The leader's commitment decision can be observed by the followers, while his amount of contribution can not be observed. Therefore, the leader's commitment decision is the only signal that transmits his information to his followers. The leader sets his contribution level $X_0^*(\alpha)$ as explained in the complete information scenario. For his commitment strategy, however, he takes into account the effect of his action on his followers' contribution level. Followers update their prior beliefs given the leader's commitment signal and react optimally to his commitment choice according to their posterior beliefs about α.

The leader makes a costly commitment to the project if his gain from the followers' contribution is more than his commitment cost. The following proposition represents this fact by specifying the Leader's **Commitment Threshold** (α^*): the smallest value of α above which the leader is willing to make a commitment to the public project.

Proposition 1 *For any equilibrium,* $\left(X_0^*, X_f^*, C^*\right)$, *there exists a threshold* $\alpha^* \in [0, \widetilde{\alpha}]$*, such that*

$$C^*(\alpha) = 1 \qquad \textit{for all } \alpha > \alpha^*$$

$$C^*(\alpha) = 0 \qquad\qquad for\ all\ \alpha < \alpha^*$$

See the appendix for the proof.

Introducing α^* simplifies the leader's commitment signal: the leader's commitment choice simply reveals whether α is higher or lower than α^*. As one can see, this signal reveals some information about α but does not reveal its exact value.

Consider a representative follower. A follower, who receives a commitment signal ($C(\alpha) = 1$), will infer that $\alpha > \alpha^*$. Therefore, his conditional expected utility given the leader's commitment is:

$$E(V_f(x_0, x_1, ..., x_m) \mid \alpha > \alpha^*) = E(\alpha \mid \alpha > \alpha^*)\sum_{j=0}^{n} x_j + U(w - x_f)$$

Recall that $\widetilde{\alpha} = U^{'}(w)$. Clearly, if $E(\alpha \mid \alpha > \alpha^*) > \widetilde{\alpha}$, then each follower's optimal contribution level is strictly positive ($X_f^*(1) > 0$) and is the unique solution to the following first order condition:

$$E(\alpha \mid \alpha > \alpha^*) = \frac{\partial U(w - x_f)}{\partial x_f}$$

If $E(\alpha \mid \alpha > \alpha^*) \leq \widetilde{\alpha}$, followers optimal contribution is zero ($X_f^*(1) = 0$).

A follower, who receives a no commitment signal ($C(\alpha) = 0$), infers that $\alpha < \alpha^*$ and therefore decides not to contribute to the project, for $E(\alpha \mid \alpha < \alpha^*) < \widetilde{\alpha}$ (because $\alpha^* \leq \widetilde{\alpha}$ according to proposition 1). As one can see the contribution decision of an uninformed follower depends on $E(\alpha \mid C(\alpha))$ rather than α itself. This enables a credible leader to increase the overall contribution level by affecting his followers' expectations. The next section investigates this in detail.

4 Overall Contributions and the Single Leader

This section pursues three main issues. First, even if it is costless to inform everybody about the quality of a project, it may improve cooperation to inform only one player and appoint him to be the leader. The second issue is about the credibility of the leader. The leader has an incentive to exaggerate α, for he gets a large benefit from his followers' participation. Rational followers recognize this and may refuse to follow the leader. Therefore, appointing a leader will not improve contributions unless we choose a leader who is able to convince his followers that he is transmitting the correct information (i.e., we must choose a credible leader). I will specify the condition sufficient to ensure a credible leadership. Third, cooperation improves on average if the leader is unable to credibly reveal all of his information.

This section is organized as follows. First, Proposition 2 specifies the leader's credibility condition and then Theorem 3 shows that a credible (convincing) leader improves participation on average by partial revelation of his private information.

Proposition 2 *Recall that α^* is the leader's commitment threshold. There exists a threshold $\alpha_c^* \in [0, \widetilde{\alpha}]$, such that:*

$$E(\alpha \mid C(\alpha) = 1) > \widetilde{\alpha} \qquad if\ \alpha^* > \alpha_c^*$$

$$E(\alpha \mid C(\alpha) = 1) < \widetilde{\alpha} \qquad if\ \alpha^* < \alpha_c^*$$

See the appendix for the proof.

The threshold α_c^* specified in Proposition 2, is called the **Credibility Threshold**. A leader is credible if his Commitment Threshold exceeds α_c^*. This condition simply says that the leader is not willing to make a commitment to very low return projects.

The credibility of the leader is an important consideration. Factors such as the leader's cost of commitment, the original endowment, and the number of followers are essential in determining the leader's credibility[8].

Let us assume that a credible leader exists (i.e., $\alpha^* > \alpha_c^*$) and consider the following ex-post scenarios:

Case 1) $\alpha^* < \alpha \leq \widetilde{\alpha}$

Case 2) $\alpha < \alpha^* \leq \widetilde{\alpha}$

[8] Some of these factors are important when it comes to selecting a leader from a heterogeneous population. See Komai and Stegeman"An Economic Theory of Leadership Based on Assignment of Information", Working Paper E-2004-4, Virginia Tech.

Case 3) $\alpha^* \leq \widetilde{\alpha} < \alpha$

In Case 1, players refuse to contribute under complete information. Under incomplete information, however, the leader commits to the public good project ($C(\alpha) = 1$) and his commitment is followed by the others. Therefore, appointing a credible leader improves cooperation.

In the second case, nobody cooperates under either scenario.

In case three, people contribute in both scenarios. The amount of contribution under incomplete information, however, is different than under complete information. Players' optimal contribution under complete information depends on the true value of α but followers' optimal contribution under incomplete information depends on $E(\alpha \mid C(\alpha) = 1)$. Therefore, in case 3 a credible leader improves cooperation **ex-post** only if $E(\alpha \mid C(\alpha) = 1) > \alpha$.

The ex-ante result unambiguously favors the single leader scenario. Theorem 3 states that a credible leader increases his follower's expected contribution ex-ante.

Theorem 3 *Given* $\alpha^* > \alpha_c^*$, $\underset{\alpha}{E}\left[X_f^*(C(\alpha) = 1)\right] > \underset{\alpha}{E}\left[\widetilde{X}(\alpha)\right]$

See the appendix for the proof.

According to Theorem 3, expected contributions to the public project are unambiguously higher, ex-ante, in the leader-follower setting than under complete information. The intuition behind this result is the following. In the leader-follower (incomplete information) setting, followers are effectively forced to hold fixed money balances given $\alpha > \widetilde{\alpha}$. If they have complete information, however, then their contributions depend on α and this creates variance in their money balances given that $\alpha > \widetilde{\alpha}$. The convexity of the marginal utility of money (from non-increasing absolute risk aversion) then implies that the expected marginal return to money given that $\alpha > \widetilde{\alpha}$ is higher than in the leader-follower setting. Therefore, players hold higher expected money balances under complete information given $\alpha > \widetilde{\alpha}$; moreover, they are keeping all of their money if $\alpha < \widetilde{\alpha}$. These facts imply lower expected contributions to the public good under complete information[9].

[9] I am grateful to Mark Stegeman for providing the intuition for this result.

So far I have shown that a credible leader increases the average level of contribution above the complete information level. An important issue, however, is how the leader affects the total surplus generated by the group. As mentioned in the introduction, the leader's commitment signal has two separate effects on the total surplus. On one hand, it affects the total surplus by persuading cooperation. On the other hand, it reduces the total surplus for it is costly. The surplus generated by the leader's action is the net effect of the two. In the next section, I derive the efficiency results for linear utility functions.

5 Efficiency Improvements and the Single Leader

To analyze how the leader affects the welfare of the group I assume that α is uniformly distributed and players' utility functions are linear in the overall return to the public project and consumption of the private good:

$$V(x_0, x_1, ..., x_m) = \alpha \sum_{j=0}^{m} x_j + w - x_i$$

Under complete information, players' optimal contribution level is the result of the following first order condition:

$$\frac{\partial V(x_0, x_1, ..., x_m)}{\partial x_i} = \alpha - 1 = 0$$

Therefore,

$$\begin{aligned} \widetilde{X}(\alpha) &= w, \ if \ \alpha > 1 \\ \widetilde{X}(\alpha) &= 0, \ if \ \alpha < 1 \\ \widetilde{X}(\alpha) &\in [0, w], \ if \ \alpha = 1 \end{aligned}$$

That is, under complete information, players are willing to contribute all of their endowment if $\alpha > \widetilde{\alpha} = 1$ and they have no incentive to contribute if $\alpha < \widetilde{\alpha} = 1$. The efficient outcome, however, is to make a positive contribution for $\alpha > \dfrac{1}{m+1}$.

In the single leader scenario, it is shown that the leader is able to persuade contribution by sending a costly commitment signal with, $R(\alpha) = m\theta r(\alpha)$ being the cost of commitment. As shown in the previous sections, there exists an equilibrium in which the leader commits to the project and is followed by the others if the following conditions hold:

$$\alpha > \alpha^* = \frac{\theta r(\alpha^*)}{w}{}^{10} \tag{i}$$

$$E(\alpha \mid \alpha > \alpha^*) > 1 \tag{ii}$$

Condition (i) says that α should be higher than the leader's Commitment Threshold α^*; that is, it should be worthwhile for the leader to commit to the public project. Condition (ii) establishes a credible leadership. It states that the leader's commitment should make the followers believe that it is worthwhile to contribute[11].

A credible leader, therefore, is able to induce participation for all values of $\alpha \in (\alpha^*, 1)$ at which nobody is willing to participate under complete information. For $\alpha > 1$ and $\alpha < \alpha^*$ both scenarios are the same.

Let us compare the welfare level in these two scenarios. For $\alpha > \widetilde{\alpha}$, the ex-post welfare gain from contribution equals $\triangle W(\alpha) = (m+1)[(m+1)\alpha - 1]w$, which is positive and equal in both scenarios. For $\alpha \in (\alpha^*, \widetilde{\alpha})$ players do not contribute under complete information. The ex-post welfare gain from contributions in the single leader scenario, however, is $\triangle W_I(\alpha) = m[(m+1)\alpha - 1]w - R(\alpha)$ which is positive for all values of α that satisfy the inequality $\alpha > \dfrac{R(\alpha)}{m(m+1)w} + \dfrac{1}{m+1}$. Proposition 4 specifies the condition that guaranties a positive ex-post welfare gain induced by a credible leader.

Proposition 4 *If the leader is credible and* $R(\alpha^*) \geq w$*, then* $\Delta W_I(\alpha) > 0$ *for all* $\alpha \in (\alpha^*, 1)$.

See the appendix for the proof.

[10] Condition i states the leader's commitment decision. The leader is willing to commit to the project if $\alpha m w - m\theta r(\alpha) > 0$ or $\alpha > \dfrac{\theta r(\alpha)}{w}$.

[11] As one can see, the leader's commitment threshold decreases with w. Therefore, the leader is less likely to be credible if the original endowment is large (i.e, if he is rich). This is different from Andreoni (2004) in which rich players send a more credible signal by committing to a large initial contribution.

Proposition 4 states that the leader improves the welfare level ex-post if his commitment cost ($R(\alpha^*)$) is equal to or larger than the contribution cost (w). The intuition behind this result is that the leader acts as a representative player on behalf of the whole group if his commitment cost is at least as large as the contribution cost of his followers. In other words, $R(\alpha^*) = w$ implies that the leader's incentives are aligned with the other players and therefore he has no incentive to endorse an inefficient project which reduces the total welfare[12].

Let us suppose that the inequality $R(\alpha^*) \geq w$ is not satisfied. We can still show that the leader improves the welfare level on average (ex-ante). Proposition 5 states this result.

Proposition 5 *Let* $E(\Delta W_I(\alpha)) = \int_{\alpha^*}^{\widetilde{\alpha}} \{m[(m+1)\alpha - 1]w - m\theta r(\alpha)\} \frac{1}{\overline{\alpha}} d\alpha$ *denote the expected welfare gain induced by the leader in equilibrium. Then,* $E(\Delta W_I(\alpha)) > 0$*.*

See the appendix for the proof.

As mentioned before, a key feature of this model is that the leader is unable to credibly reveal all of his information, for his exact amount of contribution can not be precisely verified by his followers. The leader's information therefore is transmitted only by his commitment signal which partially reveals his information. As proposition 5 stated partial revelation of information by a credible leader increases the welfare of the group on average in spite of the cost associated with his signal.

The following section concludes the paper and discusses the further extensions.

6 Conclusion

A standard result of the theory of the public goods is that public goods are underprovided by voluntary contributions.

This paper introduces leadership as an institutional solution to the free-riding problem. In my model, a leader is a person who has exclusive information about the value of the public project.

[12] This implies that a good leader (who maximizes the total surplus ex-post) is someone whose incentives are alligned with an average player.

I developed a model of public goods provision with fairly general assumptions under two different scenarios: (a) complete information and (b) incomplete information with a single leader.

In the complete information scenario all players are informed about the quality of the project and simultaneously decide how much to contribute.

In the single leader scenario, one player (the leader) is exogenously informed about the return to the public project. The leader is able to partially (not fully) transmit his information to the others by making a costly commitment. Comparing these two cases, I draw the following conclusions:

I show that by sending a vague rather than a precise signal a credible leader induces cooperation in cases where cooperation is efficient but can not be obtained under complete information.

It is shown that a credible leader can improve the average level of contribution by partial (not full) revelation of his information. That is partial revelation of information induces more cooperation ex-ante than full revelation or complete information.

The word credible refers to a leader who is able to convince his followers that he is transmitting the correct information. I specify the condition under which the leader is credible enough to induce a following.

Assuming linearity in the utility functions, I show that a credible leader increases the welfare level on average (ex-ante). I also show that a credible leader can increase the welfare ex-post if his commitment cost is equal or larger than the contribution cost of the other players. Such a leader acts as a representative player on behalf of the group and endorses the public project when it is efficient to do so.

Komai (2002)[13], and Komai and Stegeman (2004)[14] introduce the idea that partial revelation of a leader's information can produce **first best** outcomes. The distinctive feature of these models is that the leader's contribution is discrete and observable,

[13] Komai, M. (2002) "Sequential Public Goods Provision Under Incomplete Information", *Working Paper no. E-2002-8*, Virginia Tech, Page 19.

[14] Komai, M., Stegeman, M. "An Economic Theory of Leadership Beased on Assignment of Information", *Working Paper no. E-2004-4*, Virginia Tech.

and therefore there is no separate commitment decision[15]. The elimination of the unproductive signal allows full efficiency[16].

In Komai and Stegeman (2004) partial revelation of information by the leader not only solves the free riding problem but also eliminates the **coordination** problem among the players. They also show that if players are differentiated, then the optimal leader should be either average or less cooperative than the average player, and it is never optimal to appoint an unusually energetic leader. They also analyze the possibility of solving the leader's credibility problem by dividing authority among several leaders[17].

[15]The model in Komai and Stegeman (2004) is somewhat complementary to the model in the current paper in the sense that they assume that decisions are binary but generalize the specification of preferences.

[16]The first draft of Komai (2002) made this point in the context of the linear payoff function $\alpha \sum_{j=0}^{m} x_j + w - x_i$.

I am grateful to Lise Vesterlund for her encouraging reply to that draft and for subsequently serving on [author's] dissertation committee.

Potters, Sefton, and Vesterlund (2004) test the linear case experimentally, for a two-player game. They add a prior voting stage in which the two players must simultaneously affirm that the informed player will contribute first; otherwise the players contribute simultaneously. The experimental results broadly confirm the predictions of the model, in the sense of supporting the efficient equilibrium.

[17]The multileader case appears in a direct antecedent of the 2004 paper, with the same title, at http://www.hss.caltech.edu/~jacksonm/dukeprogram.htm

7 Appendix

Proof of Proposition 1:

By assumption $r(\alpha) = 0$ for all $\alpha > \widetilde{\alpha}$. Therefore $C(\alpha) = 1$ for all $\alpha > \widetilde{\alpha}$.

Let $h(\alpha) = \alpha m \left(X_f^*(1) - X_f^*(0)\right) - m\theta r(\alpha)$ denote the leader's utility gain from the followers' contribution if he commits to the project. Clearly, $h(\alpha) > 0$ implies $C(\alpha) = 1$ and $h(\alpha) < 0$ implies $C(\alpha) = 0$.

If $C(\alpha) = 0$ for all $\alpha \in [0, \widetilde{\alpha})$, then $\alpha^* = \widetilde{\alpha}$.

Suppose $C(\widehat{\alpha}) = 1$ for some $\widehat{\alpha} \in [0, \widetilde{\alpha})$. Then $h(\widehat{\alpha}) \geq 0$, implying that $\left(X_f^*(1) - X_f^*(0)\right) > 0$. We also know by assumption that $r'(\alpha) < 0$. Thus we can conclude that $h'(\alpha) > 0$ for all $\alpha > \widehat{\alpha}$. This implies that $h(\alpha) > 0$ for all $\alpha > \widehat{\alpha}$. Therefore $C(\alpha) = 1$ for all $\alpha > \widehat{\alpha}$.

Summarizing: $C(\widehat{\alpha}) = 1$ for some $\widehat{\alpha} \in [0, \widetilde{\alpha})$, implies $C(\alpha) = 1$ for all $\alpha > \widehat{\alpha}$. Pick $\alpha^* = Inf\left\{\alpha : C(\alpha) = 1\right\}$.

Proof of Proposition 2:

For this proof, let α^* denote the leader's choice of contribution threshold without requiring that this be an equilibrium choice.

First, I prove fact (1): $\dfrac{\partial E(\alpha \mid C(\alpha) = 1)}{\partial\ \alpha^*} > 0$.

Proof of fact (1):

$$E(\alpha \quad | \quad C(\alpha) = 1) = E(\alpha \mid \alpha \geq \alpha^*) = \int_{\alpha^*}^{\overline{\alpha}} \alpha f(\alpha \mid \alpha \geq \alpha^*) d\alpha$$

$$= \int_{\alpha^*}^{\overline{\alpha}} \frac{\alpha f(\alpha)}{1 - F(\alpha^*)} d\alpha = \frac{1}{1 - F(\alpha^*)} \int_{\alpha^*}^{\overline{\alpha}} \alpha f(\alpha) d\alpha$$

Taking the derivative of $E(\alpha \mid \alpha \geq \alpha^*)$ with respect to α^* we have:

$$\begin{aligned}
\frac{\partial E(\alpha \mid \alpha \geq \alpha^*)}{\partial\ \alpha^*} &= \frac{f(\alpha^*)}{[1-F(\alpha^*)]^2}\int_{\alpha^*}^{\overline{\alpha}} \alpha f(\alpha) d\alpha - \frac{\alpha^* f(\alpha^*)}{1-F(\alpha^*)} \\
&= \frac{f(\alpha^*)}{1-F(\alpha^*)}\int_{\alpha^*}^{\overline{\alpha}} \frac{\alpha f(\alpha)}{1-F(\alpha^*)} d\alpha - \frac{\alpha^* f(\alpha^*)}{1-F(\alpha^*)} \\
&> \frac{f(\alpha^*)}{1-F(\alpha^*)}\int_{\alpha^*}^{\overline{\alpha}} \frac{\alpha^* f(\alpha)}{1-F(\alpha^*)} d\alpha - \frac{\alpha^* f(\alpha^*)}{1-F(\alpha^*)} \\
&= \frac{\alpha^* f(\alpha^*)}{1-F(\alpha^*)}\left[\int_{\alpha^*}^{\overline{\alpha}} \frac{f(\alpha)}{1-F(\alpha^*)} d\alpha - 1\right] = \frac{\alpha^* f(\alpha^*)}{1-F(\alpha^*)} \times 0 = 0
\end{aligned}$$

(Proof of fact 1 completed.)

Considering fact one,

If $E(\alpha \mid C(\alpha) = 1) < \widetilde{\alpha}$ for all $\alpha^* \in [0, \widetilde{\alpha}]$, then $\alpha_c^* = \widetilde{\alpha}$.

If $E(\alpha \mid C(\alpha) = 1) > \widetilde{\alpha}$ for all $\alpha^* \in [0, \widetilde{\alpha}]$, then $\alpha_c^* = 0$.

If $E(\alpha \mid C(\alpha) = 1) = \widetilde{\alpha}$, let $\alpha_c^* = \alpha^*$.

Proof of Theorem 3:

Recall that $\widetilde{X}(\alpha)$ is player i's optimal contribution under complete information. Thus the first order condition implies that :

$$\alpha = U'(w - \widetilde{X}(\alpha)) \tag{i}$$

Also recall that $X_f^*(1)$ is follower f's optimal contribution in the single leader scenario if the leader endorses the project. Thus the first order condition implies that:

$$E(\alpha \mid C(\alpha) = 1) = U'(w - X_f^*(1)) \tag{ii}$$

Since $U''' > 0$, Jensen's inequality implies that: $U'\left[E\left(Z\right)\right] < E\left[U'(Z)\right]$ for any random variable Z. Therefore,

$$U'\left[E\left(w - \widetilde{X}(\alpha)|C(\alpha) = 1\right)\right] < E\left[U'(w - \widetilde{X}(\alpha))\ |C(\alpha) = 1\right] \tag{iii}$$

Since $\alpha^* \leq \widetilde{\alpha}$ we have:

$$E\left[\widetilde{X}(\alpha)|C(\alpha)=0\right]=0$$

Thus:

$$\begin{aligned} E\left[\widetilde{X}(\alpha)\right] &= \Pr(\alpha<\alpha^*).E\left[\widetilde{X}(\alpha)|C(\alpha)=0\right]+\Pr(\alpha\geq\alpha^*).E\left[\widetilde{X}(\alpha)|C(\alpha)=1\right] \\ &= \Pr(\alpha\geq\alpha^*).E\left[\widetilde{X}(\alpha)|C(\alpha)=1\right] \qquad \text{(iv)} \end{aligned}$$

(i) and (iii) imply that:

$$U'\left[E\left(w-\widetilde{X}(\alpha)|C(\alpha)=1\right)\right] < E(\alpha \mid C(\alpha)=1)$$

Using (ii) we have:

$$U'\left[E\left(w-\widetilde{X}(\alpha)|C(\alpha)=1\right)\right] < U'(w-X_f^*(1))$$

From $U''<0$ it follows that:

$$w-E\left(\widetilde{X}(\alpha)|C(\alpha)=1\right) > w-X_f^*(1)$$

or

$$E\left(\widetilde{X}(\alpha)|C(\alpha)=1\right) < X_f^*(1)$$

Thus,

$$\Pr(\alpha\geq\alpha^*).E\left(\widetilde{X}(\alpha)|C(\alpha)=1\right) < \Pr(\alpha\geq\alpha^*).X_f^*(1) \qquad \text{(v)}$$

Therefore from (iv) and (v) we have:

$$E\left[X_f^*(1)\right] > E\left[\widetilde{X}(\alpha)\right]$$

Proof of Proposition 4:

The inequality $R(\alpha^*) \geq w$ implies that $m\theta r(\alpha^*) \geq w$. Adding $\theta r(\alpha^*)$ to both sides of the inequality I have:

$$(m+1)\theta r(\alpha^*) - w - \theta r(\alpha^*) \geq 0$$

If I multiply and divide the first term by w I get:

$$\frac{w(m+1)\theta r(\alpha^*)}{w} - w - \theta r(\alpha^*) \geq 0$$

Since according to equation i, $\alpha^* = \dfrac{\theta r(\alpha^*)}{w}$, I can rewrite the above inequality as:

$$(m+1)w\alpha^* - w - \theta r(\alpha^*) \geq 0$$

Multiplying both sides by m I get:

$$m\left[(m+1)\alpha^* - 1\right]w - m\theta r(\alpha^*) \geq 0$$

The above inequality simply shows that $\Delta W_I(\alpha) \geq 0$ at $\alpha = \alpha^*$. Since $r' < 0$, $\Delta W_I(\alpha)$ is an increasing function of α. Therefore $\Delta W_I(\alpha) \geq 0$ at $\alpha = \alpha^*$, implies that $\Delta W_I(\alpha) > 0$ for any $\alpha \in (\alpha^*, \widetilde{\alpha})$.

Summarizing: $R(\alpha^*) \geq w$ implies $\Delta W_I(\alpha) > 0$ for any $\alpha \in (\alpha^*, \widetilde{\alpha})$.

Proof of Proposition 5:

Choose $\theta' > 0$ such that $r(\alpha^*) = \theta'(\widetilde{\alpha} - \alpha^*)$ and define

$$A = \int_{\alpha^*}^{\widetilde{\alpha}} \left\{m\left[(m+1)\alpha - 1\right]w - m\theta\theta'(\widetilde{\alpha} - \alpha)\right\}\frac{1}{\widetilde{\alpha}}d\alpha$$

$$A = \int_{\alpha^*}^{\widetilde{\alpha}} \left[m(m+1)\alpha w - mw - m\theta\theta'\widetilde{\alpha} + m\theta\theta'\alpha\right]\frac{1}{\widetilde{\alpha}}d\alpha$$

$$
\begin{aligned}
A &= \left(m^2 w + mw + m\theta\theta'\right) \int_{\alpha^*}^{\widetilde{\alpha}} \alpha \frac{1}{\overline{\alpha}} d\alpha - (mw + m\theta\theta'\widetilde{\alpha}) \int_{\alpha^*}^{\widetilde{\alpha}} \frac{1}{\overline{\alpha}} d\alpha \\
A &= \left(m^2 w + mw + m\theta\theta'\right) \left[\frac{\alpha^2}{2\overline{\alpha}}\right]_{\alpha^*}^{\widetilde{\alpha}} - (mw + m\theta\theta'\widetilde{\alpha}) \left[\frac{\alpha}{\overline{\alpha}}\right]_{\alpha^*}^{\widetilde{\alpha}} \\
A &= \left(m^2 w + mw + m\theta\theta'\right) \left[\frac{w^2 + 2w\theta\theta'}{2\overline{\alpha}(w^2 + \theta^2\theta'^2 + 2w\theta\theta')}\right] - \left(\frac{mw}{\overline{\alpha}}\right) \\
A &= \frac{m^2 w^3 + 2m^2 w^2 \theta\theta' - mw^3 - mw^2\theta\theta'}{2\overline{\alpha}(w^2 + \theta^2\theta'^2 + 2w\theta\theta')}
\end{aligned}
$$

As it can be seen, A is positive for $m > 1$. By assumption $r(\alpha)$ is a convex function. This implies that $E(\Delta W_I(\alpha))$ is a concave function of α. Therefore, $E(\Delta W_I(\alpha)) > A > 0$.

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