Let \( \emptyset, \lambda \) and \( a \) denote the regular expressions that respectively denote the empty set, the set \( \{\lambda\} \) and the set \( \{a\} \), where \( \lambda \) is the empty string and \( a \) is a letter appearing in the underlying alphabet. Further, let \( r, s \) and \( t \) denote arbitrary regular expressions.

**Identities of Regular Expressions**

**Basic identities**

1. \( \emptyset + r = r = r + \emptyset \) \( \emptyset \) is the identity for union.
2. \( \lambda r = r = r \lambda \) \( \lambda \) is the identity for concatenation.
3. \( \emptyset r = \emptyset = r \emptyset \) \( \emptyset \) is the annihilator for concatenation.
4. \( r + r = r \) Idempotence law for union.
5. \( r + s = s + r \) Commutative law for union.
6. \( (r + s) + t = r + (s + t) \) Associative law for union.
7. \( (rs)t = r(st) \) Associative law for concatenation.
8. \( (r + s)t = rt + st \) Distributive law of concatenation over union.

**Next level of identities**

9. \( \emptyset^* = \lambda \)
10. \( \lambda^* = \lambda \)
11. \( rr^* = r^*r \)
12. \( \lambda + rr^* = r^* \)
13. \( (\lambda + r)^* = r^* \)
14. \( r(sr)^* = (rs)^* r \)
15. \( (rs + r)^* r = r(sr + r)^* \)
**Not-so-obvious identities**

16. \( r^*r^* = r^* \)

17. \( (r^*)^* = r^* \)

18. \( (r^*s^*)^* = (r+s)^* \) \( (s^*r^*)^* = (r+s)^* \)

19. \( (r^* + s^*)^* = (r+s)^* \)

20. \( r^*(s r^*)^* = (r+s)^* \) \( (r^* s^*)^* r^* = (r+s)^* \)

21. \( (r^* s^*)^* = \lambda + (r+s)^* s \) \( (r s^*)^* = \lambda + r(r+s)^* \)