Fall 2003, Practice NCS-MAA exam.

1) **The Truth is Out There**

Suppose five percent of the population are actually Bug-Eyed Monsters (BEMs), sent here to spy on us. The government comes up with a test for BEMs, and the test works almost all the time: when a BEM is tested, 98 percent of the time the test will identify the monster. Unfortunately, the test sometimes doesn't work: 3 percent of humans tested are also identified as monsters. What percentage of those who tested positive are humans?

2) **I Palindrome I**

Call a number a "mirror number" if it is symmetric about its center. 121 and 1221 are both mirror numbers; 182 and 2333 are not. Are mirror numbers with an even number of digits divisible by 11? Are mirror numbers with an odd number of digits divisible by 11?

3) **Valhalla**

Odin and the Norse Heroes were feasting in the Great Hall of Valhalla, which holds up to 5000 soldiers. Yet the Hall was more than half empty, and Odin summoned the bard Snorri. "Snorri", says Odin, "how many Heroes are here?" And Snorri thought a moment, and had all present pair up, and then sit down, and a sole Hero was left standing. And Snorri did the same in groups of three, and five, and seven, and eleven, and each time only 2 Heroes were left standing. And then Snorri told Odin how many Heroes were present, and Odin was sore wroth. How many Heroes were there?

4) **The Ubiquitous Floor Function**

Remember that $\lfloor x \rfloor$ is the greatest integer less than or equal to x. Find the area between the graphs of $f(x) = x^2 - (1/2)x$ and $g(x) = x\lfloor x \rfloor$ from $x = 0$ to $x = 24036$.

5) **This Year's Term**

Show that $1^{2003}/2 + 2^{2003}/2^2 + 3^{2003}/2^3 + 4^{2003}/2^4 + \cdots$ is an integer.

6) **Given to my Calc I students this day as extra credit**

Consider a function $y = f(x)$ with the following properties. $f(x)$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$; $f(0) = 0$; and $|f'(x)| \leq |f(x)|$. What is $f(x)$?

7) **The Tex Winter Problem**

Let $p(x)$ be a polynomial, with two distinct real critical numbers, and three distinct roots. Show that either the roots are collinear with the critical numbers, or the roots form a triangle in the complex plane, and the two critical numbers lie within the triangle.

8) **From the 1946 Putnam**

Show that the next integer above $(\sqrt{3} + 1)^{2n}$ is divisible by 2^{n+1}

9) **Ran out of clever names**

Given any $n + 1$ distinct integers between 1 and $2n$, show that two of them are relatively prime.

10) What does NCS-MAA stand for?