

MAA NCS Practice Problems Set 2
October 29, 2004

1) Suppose $f(x)$ is a non-constant differentiable function defined on $[0, 1]$ with $f(0) = 0$ and $|f'(x)| \leq |f(x)|$. Show $f(x) = 0$, for all $x \in [0, 1]$.

2) If a_0, a_1, \dots, a_n are real numbers satisfying

$$\frac{a_0}{1} + \frac{a_1}{2} + \dots + \frac{a_n}{n+1} = 0$$

show that the equation $a_0 + a_1x + a_2x^2 + \dots + a_nx^n = 0$ has at least one real root.

3) Find this limit:

$$\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x (1 + \sin(2t)) dt$$

A harder limit:

$$\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x (1 + \sin(2t))^{1/t} dt$$

4) Find this integral:

$$\int \frac{1}{1 + e^x} dx$$