

MAA NCS Practice Problems Set 3
November 5, 2004

- 1) How many solutions are there to the equation $|2x - 1| - |x + 5| = 3$?

- 2) Two circles of radius 1 and radius 2 are placed so they are tangent to each other and a straight line (see the picture below). A third circle is nestled between them so that it is tangent to both circles and the line. Find the radius of the third circle.

- 3) A man whose jeep has a vertical windshield drives a mile through a vertical rain consisting of drops that are uniformly distributed and falling at a constant rate. Should he go fast or slow in order to minimize the amount of rain that strikes the windshield?

- 4) Given a set of 51 points in a unit square, show that there is always a set of three of these points which lie interior to a circular disc of radius $1/7$.

- 5) Let m be a positive integer and let n be an integer obtained from m by rearranging the digits of m in some way (for example: $m = 312$ and $n = 213$). Show that $m - n$ is divisible by 9.

- 6) Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be continuous, non-constant, periodic functions. Is it possible that $h(x) = f(xg(x))$ is periodic? (This is an open question, posed in this month's MAA Monthly; if you solve it, write up a solution and send it in (by March)).