Problem Set Two Hints
Math 499, Spring 2005

A Suggestion From Kumiko and Sam. Let's meet Friday at 3pm, not Friday at 4pm, since Austin is out of town.

Problem 2. As written, the answer is "no", because lines are odd degree polynomials. I didn't do quite as good a job proofreading as perhaps I should have; consider instead odd degree polynomials of degree at least 3.

Problem 5. It was asked if the answer is:

$$a^2 \tan(\alpha/2)\left(1 - \frac{\pi}{2} + \frac{\alpha\pi}{360}\right)$$

Keep in mind that these are hints, not verification of answers. But I will point out something: our angles are measured in radians, not degrees, so that 360 looks really out of place. If the 360 is replaced with 2π, we get:

$$a^2 \tan(\alpha/2)\left(1 + \frac{\alpha}{2} - \frac{\pi}{2}\right)$$

I know that $\pi/2 \cong 1.57$; what's the sign of this formula if $\alpha = .5$?

In any case, this formula is certainly on the right track.

Problem 10. There are some definitional problems here. A *proper divisor* of a positive integer is any divisor of the integer, except the integer itself. So the proper divisors of 6 are $1, 2, 3$; the proper divisors of 10 are $1, 2, 5$; the proper divisors of 28 are $1, 2, 4, 7, 14$. Both 6 and 28 are perfect numbers ($1+2+3 = 6$ and $1+2+4+7+14 = 28$) but 10 is not ($1+2+5 = 8 \neq 10$). The lengths of the sides of a Pythagorean right triangles are all integers. Such a triangle has three sides: the hypotenuse (opposite the right angle) and two legs (the sides forming the right angle). It's known that at most one of the two legs can be a perfect number; you are asked to show that (or at least investigate the possibilities). It is not known if it is possible that one leg AND the hypotenuse can both be perfect numbers. Do some investigations; if you can find a proof you can get it published (and if you can find something that looks like it leads to a proof, I and probably other members of the department would be happy to help you over some of the rough spots).

To find out more than you ever thought possible about perfect numbers, head over to MathWorld (http://mathworld.wolfram.com), and search for "Perfect Number".