Problem Set Four Hints
Math 499, Spring 2005

Problem 1. If you have

$$M^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}$$

then you are hoping to show that

$$M^{n+1} = \begin{bmatrix} 2(n+1)+1 & -4(n+1) \\ n+1 & -2(n+1)+1 \end{bmatrix}$$

But you can calculate M^{n+1}:

$$M^{n+1} = (M^n)(M) = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

If you multiply this out, do you get the formula above for M^{n+1}?

Problem 10. Here's the formula:

$$\sum_{k=1}^{n} \frac{1}{k\binom{n}{k}} = \frac{1}{2^{n-1}} \sum_{k=1,\ k\text{odd}}^{n} \frac{\binom{n}{k}}{k}$$

Let's try this for the first three values of n and see what we get.

Let $n = 1$. Then

$$LHS = \frac{1}{1\binom{1}{1}} = 1$$

$$RHS = \frac{1}{2^0}\frac{\binom{1}{1}}{1} = 1$$

so that works.

Let $n = 2$ and note that the only odd number less than 2 and greater than 0 is 1. But let's include the second line of Pascal's triangle here:

$$\binom{2}{0} = 1 \quad \binom{2}{1} = 2 \quad \binom{2}{2} = 1$$

$$LHS = \frac{1}{1\binom{2}{1}} + \frac{1}{2\binom{2}{2}} = \frac{1}{2} + \frac{1}{2} = 1$$

$$RHS = \frac{1}{2^1}\frac{\binom{2}{1}}{1} = 1$$

so that also works.

Let $n = 3$; now on the right side we will have two terms, one with $k = 1$ and one with $k = 3$. We'll include the third line of Pascal's triangle:

$$\binom{3}{0} = 1 \quad \binom{3}{1} = 3 \quad \binom{3}{2} = 3 \quad \binom{3}{3} = 1$$

$$LHS = \frac{1}{1\binom{3}{1}} + \frac{1}{2\binom{3}{2}} + \frac{1}{3\binom{3}{3}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{3} = \frac{5}{6}$$

$$RHS = \frac{1}{2^2}\left(\frac{\binom{3}{1}}{1} + \frac{\binom{3}{3}}{3}\right) = \frac{1}{4}\left(3 + \frac{1}{3}\right) = \frac{3}{4} + \frac{1}{12} = \frac{10}{12} = \frac{5}{6}$$

So that works too.

I'm not entirely sure if this is induction; it looks like it should be. One thing to keep in mind is that 2^{n-1} is the sum of the $(n-1)$st row of Pascal's triangle.