Problem Set Five Hints
Math 499, Spring 2005

1) There are a number of ways to show that an integral converges or diverges. A direct comparison would certainly work: if $1/\lfloor x^2 \rfloor \leq f(x)$, and $\int_1^\infty f(x)dx$ converges, then $\int_1^\infty 1/\lfloor x^2 \rfloor dx$ converges as well. One could also find a formula for $\int_1^b 1/\lfloor x^2 \rfloor dx$, and see what the limit is as $b \to \infty$. The formula from the first part may be useful in this context. Finally, whenever dealing with an integral, it's always a good idea to sketch the graph of the function you're trying to integrate. If you do that, you'll at least get a visual impression of the area you're trying to find.

2) "Closed form" means a formula without integration, differentiation, or summation involved.

4 and 6) There's a similar hint for both of these, which is: write the function involving the floor function as a piecewise defined function.

For example, let $F(n)$ be the function on the left side in problem 4. There are two formulas for $F(n)$ that are fairly immediate:

$$F(n) = \lfloor \sqrt[4]{1} \rfloor + \cdots + \lfloor \sqrt[4]{n} \rfloor$$
$$F(n) = \sum_{i=1}^{n} \lfloor \sqrt[4]{i} \rfloor$$

The problem is asking for the intersections of $F(n)$ and the function $h(n) = 2n$. The two formulas above are not particularly useful in solving this problem, so can we find a more useful formula? And we can:

$$F(n) = \begin{cases} n & 1 \leq n < 16 \\ 2n - 15 & 16 \leq n < 81 \\ \cdots & \cdots \end{cases}$$

With a little work, you can find formulas that work for different intervals of n. Which then allows us to answer the question: for $1 \leq n < 16$, we have $F(n) = n$, and so does $F(n) = 2n$ for any n in this interval? And the answer is no. And then do the same for all the rest of the intervals.

In problem 6, we have $g(x) = x\lfloor x \rfloor$. This function can also be re-written as a piecewise defined function:

$$g(x) = \begin{cases} 0 & 0 \leq x < 1 \\ x & 1 \leq x < 2 \\ 2x & 2 \leq x < 3 \\ \cdots & \cdots \end{cases}$$

For each interval, there's a different formula for $g(x)$; using these formulas, can you find the points of intersection of $g(x)$ and $f(x)$ in that particular interval? Again, a picture might be useful here; pictures are almost always useful when trying to do integration problems.

8) The units digit is the rightmost digit in an integer: the units digit of 312459 is 9, for example.